

Personal Computer Assisted Tutorial for Smith Charts

Ron Pieper[†] and Fritz Dellsperger[‡]

[†]Department of Electrical and Computer Engineering
Naval Postgraduate School, Monterey, CA 93943

email: pieper@ece.nps.navy.mil

[‡]University of Applied Sciences Berne
Berne Institute of Engineering and Architecture

email: fritz.dellsperger@hta-be.bfh.ch

Abstract

A recognized advantage associated with the use a Smith Chart is that it obviates the employment of complex algebra in solving transmission line problems. The main point of the paper is to focus on combining the power of the PC, with its visual aesthetics, speed, and accuracy, for the purpose of familiarizing students in the use of the A. Smith's graphical transmission line calculator. The demo version of the PC Smith Chart program, which was obtained over the internet, is limited to less than 6 data point. Despite the demo version's limitations for purposes of extensive design problems it provides an excellent tool for the purposes of introductory level instruction for the Smith Chart. The collection of topics starts with the basic VSWR interpretation for a mismatched line. Rules covering impedance transformations for various point on the line are easily tested and visualized. Specifically, this will include quarter wave matching, impedance matching using transformers and stub tuning.

1. Introduction

A extensive body of literature exists which demonstrates that transmission line models are applicable to broad selection of physically diverse problems. This includes but not necessarily limited to heat transfer[1], acoustics [2], electro-magnetic plane waves at boundaries [3], waveguides, microwave devices [4,5] and of course electrical transmission lines. Within the scope of technical curricula it is possible to find the topic of transmission lines covered either in an electromagnetics course [3] or possibly in an electronic communications class[6]. For most electrical communication applications the transmission line problems come up in the context of transmission line to antenna coupling. In the case of

electrical transmission lines the interest in getting a high percentage of the signal power to the load is usually a first concern. The advantage in using a Smith Chart is that it obviates the need to employ complex algebra. For many design problems the time savings can be significant. Despite the general tendency to supplant procedures done normally by hand with computer calculations the Smith Chart continues to be a commonly employed design tool.

The demo version of the PC Smith Chart program, which was obtained over the internet[7], is limited to relatively simple transmission line problems. Despite its limitations it provides an excellent tool for the purposes of introductory level instruction for the Smith Chart. The commercially available version has greatly expanded capabilities which is suitable for a wider range of tasks which includes sophisticated professional engineering design. Nonetheless, the Windows based demo version of the program can provide the capability of data entry for 5 data points via either mouse or keyboard. The Smith Chart occupies a major part of the screen. Contours corresponding to "moving down the line" from the load are automatically generated on the screen. The coupling selections include both series and shunt options for resistance, capacitance, inductance and lossless transmission lines. Also, one can select an impedance transformer.

One can find many sources which explain from first principles a mathematical description for the contours corresponding the constant resistance or reactance.[3,6] The focus of the paper will be on the operation applications not the theoretical underpinnings for the creation of the Smith Chart.

2. T-Line basics

The transmission line is represented Fig 1. The parallel lines represents a transmission line of length

D and characteristic impedance Z_0 , a load Z_L is shown attached on the right hand side. The reflection coefficient, which is by definition the ratio of the reflected wave to the incident wave [4], can be predicted from the rule:

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} \quad (1)$$

Transmission lines tend to be lossy if operated in a frequency domain in which the characteristic impedance is complex. It will be assumed that the characteristic impedance is real for the purposes of illustrating the applications to be discussed. The impedance looking into a lossless transmission line from the source side can be predicted from the general rule:

$$Z_{in}(D) = Z_0 \frac{1 + \Gamma e^{-j2\beta D}}{1 - \Gamma e^{-j2\beta D}} \quad (2)$$

where the phase constant $\beta=2\pi/\lambda$ is dictated by the operating frequency and the velocity factor on the line. The input impedance Eq(2) will cycle between having a reactance that is alternately inductive and capacitive.

The standing wave ratio (SWR) defines the ratio of the maximum voltage to the minimum voltage in the composite signal on the line. The SWR ranges from one, which corresponds to zero reflected power, to infinity which occurs for 100% reflected power. The SWR is conveniently calculated from the rule

$$SWR = \frac{1 + |\Gamma|}{1 - |\Gamma|} \quad (3)$$

On a lossless transmission line the SWR will not change with distance from the load.

For practical reasons of providing power to the load a mismatch between line characteristic impedance and load is usually undesirable. A solution based on a quarter-wave section of transmission line can be employed when flexibility of modifying either load or line is not practical and the difference in impedance is significant. Besides the length requirement for this solution it is well known that the characteristic impedance must satisfy the rule:

$$Z'_0 = \sqrt{Z_0 Z_L} \quad (4)$$

The caveat issued is that in order for the quarter-wave impedance rule Eq (5) to predict a real impedance the load must also be real (and positive). For a broad range of engineering problems, for example antenna design, this requirement for the load is not a practical restriction. This last hurdle can be taken care of by inserting, as represented in Fig 2, an additional piece of transmission line that will transform the complex impedance such that the impedance looking into the

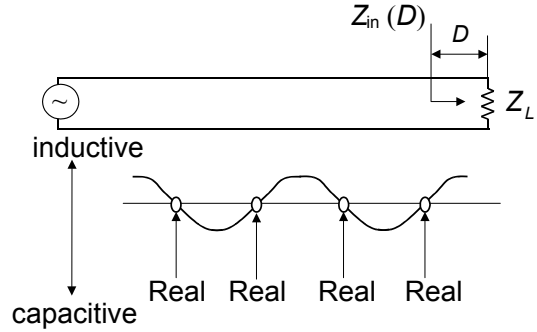


Figure 1: Transmission line model

line from the source side is real. For either a series or shunt insertion the rule based on Eq(3) is applicable. Eq(5) can be then be used to predict the quarter wave match impedance following the *principle* that the load impedance is replaced with the real quantity $Z'_{in}(D)$. In the case of shunt line the standard options are termination as either a short or an open. The Smith Chart serves as a calculator for Eq(1-3).

In some situations it is not unusual to find that a balanced to unbalanced (balun) connector can also serve as impedance transformer for a match between lines, e.g. 75 Ω coax connecting to 300 twin lead or 300 Ω folded dipole antenna to 75 Ω cable. The impedance transformer rule is

$$Z_{in} = n^2 Z_L \quad (5)$$

where n is the turns ratio on the input side divided by the turns ratio on the load side.

3. Smith Chart Basics

The Smith Chart provides a direct reading of impedance consistent with Eq(2). As a disclaimer it is worth noting that the Smith Chart program has the ability display and predict both impedance and admittance values. Due to limited space the focus here is to illustrate Smith Chart methods using only the impedance curves. Fig 3, was extracted from an image generated from the Smith Chart program[5]. Constant resistance contours are observable on the chart as expanding circles, while the constant reactance contours are observable as sets of hyperbolas. The hyperbolas above(below) the horizontal line correspond to positive(negative) reactance(inductive) and exhibiting inductive (capacitive) properties. The sum of the real part and the imaginary part form the predicted impedance. As predicted from Eq(1) a mismatch between load and line will lead to a nonzero reflection coefficient and a SWR greater than one. Observe on Fig 3 that for a load of 100 Ω and a transmission line characteristic

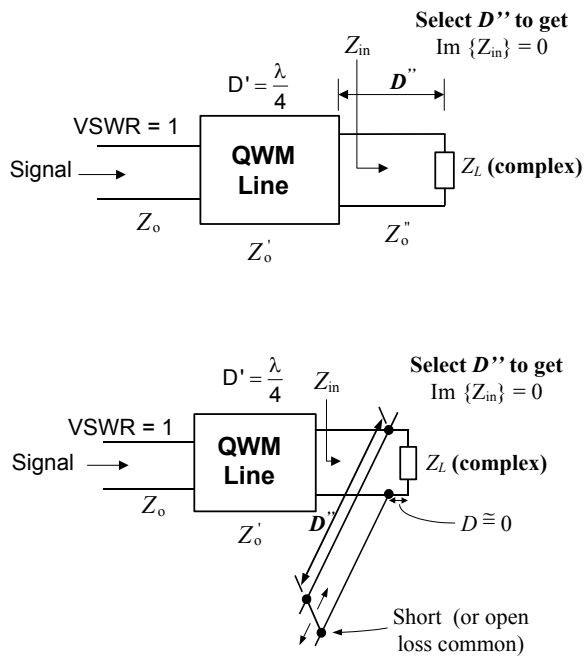


Figure 2: Two solutions with complex loads; (a) series above and (b) shunt below.

impedance of 50Ω a constant SWR circle is shown. This curve on the graph was obtained by first selecting the load impedance and then selecting in series the transmission line's characteristic impedance.

The mouse driven cursor of the program will then be constrained to points on the circle. Assuming that the load was chosen such that no mismatch occurs the SWR circle would have degenerated to essentially a point at the characteristic impedance. Movement of the mouse then allows for selection of line termination distance D . The arrow on the SWR circle gives the direction (clockwise) for increasing distance from the load. To illustrate, the transmission line distance, $D \approx 0.25 \lambda$, was selected. Once selected the cursor is again free to roam over the full face of the chart. A full screen capture taken in which the cursor is on top of data point 2 is shown in Fig 4. Note, the program provides at the bottom of the screen numerical values for the impedance (25Ω), the reflection coefficient (-0.33) and the standing wave ratio (2). The characteristic impedance is also indicated at the bottom on the left hand side to be 50Ω .

The options for modification of this value are limited in the demo version but this is not a serious restriction if the main objective is to illustrate simple methodology. One caveat is that the values for reflection coefficient and SWR are then valid for a 50Ω characteristic impedance line.

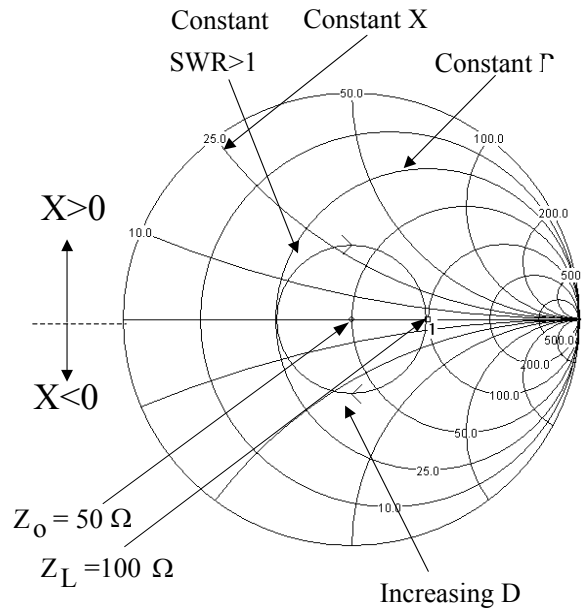


Figure 3: Smith Chart basics

4. Selected Examples

The three examples discussed here were employed in a classroom setting. The students did not have prior exposure to Smith Chart mechanics. These examples demonstrate the power of the Smith Chart PC program to facilitate these operations.

Example 1 (Quarter Wave Match, complex load, series solution)

In this example a quarter wave match *series* T-line solution is obtained for a complex inductive load. The problem was posed to the students as represented by Fig 2a where the students are informed to assume that the complex load impedance is $(50+50j)\Omega$ and the series transmission line used between quarter wave section and load has an arbitrarily selected characteristic impedance of $Z_o''=75 \Omega$. The problem specifies that the characteristic impedance of the transmission line connected to the source has a $Z_o=50\Omega$ and it is desired that the solution exhibits a unity SWR on this line. The steps taken with the Smith Chart program will indicate that two solutions are possible however in order to keep the discussion brief only one will be pursued here. Following the principle stated earlier the cursor is stopped on a point on the horizontal axis making the input impedance real. The load is the starting point and denoted on the Smith Chart program display as point 1. The intermediate solution (point 2) with $D'' \approx 0.381\lambda$ will arbitrarily be selected. The alternate rejected solution occurs at $D'' \approx 0.131\lambda$.

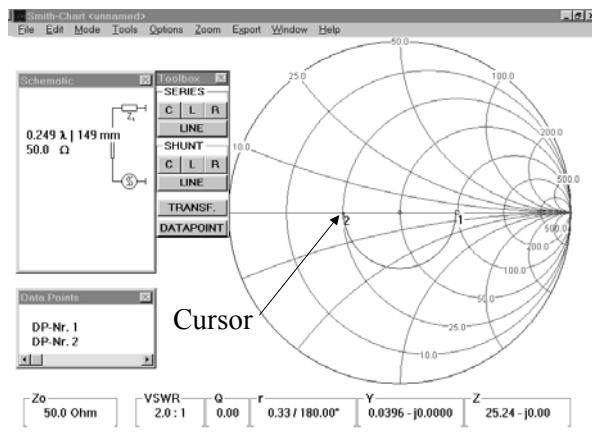


Figure 4: Mismatch between load and line

Fig 5, which is a screen capture from the Smith Chart program, represents the final solution. The input impedance at point 2 was recorded to be approximately 31.25Ω . At this point it is necessary to predict via calculator an appropriate quarter wave section characteristic impedance use Eq(5). $Z'_o = \sqrt{50 \times 31.25} \approx 39.53 \Omega$. As expected the quarter wave section will lead to a termination on approximately on 50Ω at data point 3. Following this with a series transmission line of 50Ω completes the design and the SWR contour is simply a point at the impedance $Z=50\Omega$. Note that data point 4 appears on top of data point 3.

Example 2 (Impedance Transformer)

In this example the load is assumed to be a 300Ω folded dipole antenna and an impedance transformer solution is pursued in which a match to a 75Ω coax cable is desired.

For a 75Ω input the turns ratio predicted by the program is 0.5. See Fig 6. This can be confirmed from Eq(5). Assuming that in error a 50Ω coax line was substituted for the intended 75Ω an impedance mismatch is expected and the resulting SWR is 1.5. This can be confirmed from Eq(1) and Eq(3).

Example 3 (Quarter Wave Match, complex load, shunt solution)

This problem posed to the students is represented by Fig 2b. The load was arbitrarily selected to be $(50-50j) \Omega$, i.e. capacitive. Antennas that are shorter than the ideal resonant condition behave capacitively. It was assumed that the transmission line from the source has a characteristic impedance of 50Ω . The requirement on the solution is that the SWR on this line is unity. A shunt line of 75Ω impedance was

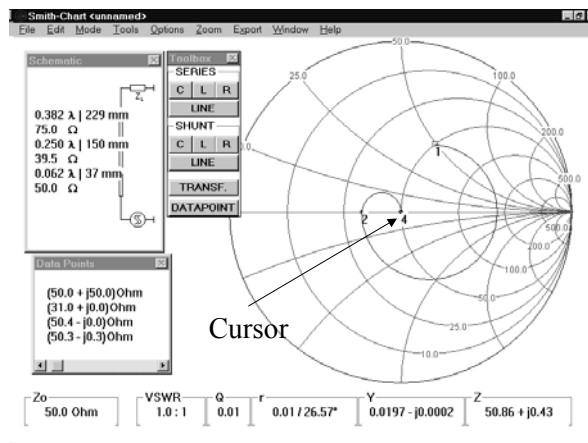


Figure 5: Complex load series solution

arbitrarily selected to tune out the reactive component. An SWR circular contour is again generated but it is noteworthy that the arrow for increasing D goes counter clockwise. There exists only one practical length for this shunt line. The case where $D''=0$ as predicted by the program will correspond to $Z=0$ and no power can be delivered to the load. The solution is to take the other case where $D'' \approx 0.148\lambda$. The Smith Chart program predicts that $Z_{in} \approx 100\Omega$. In order to match to the 50 line, it can be predicted from Eq(4) that the quarter wave section should have a characteristic impedance of 70.7Ω . The predictions for D'' and Z_{in} came directly from the Smith Chart program and obviated complex algebra required with formal solutions based on Eq(3). A screen capture for this solution is shown in Fig 7.

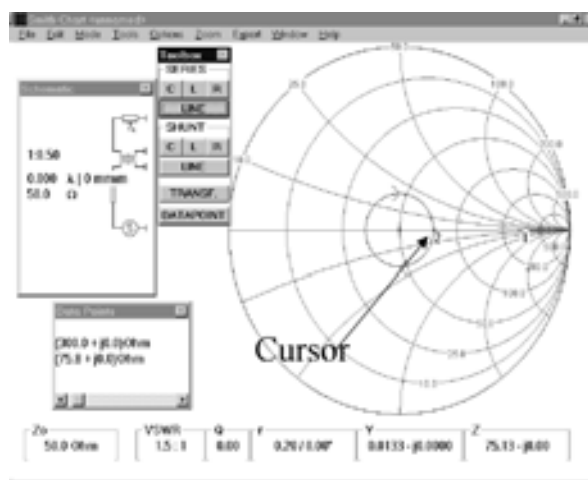


Figure 6: Impedance transformer application

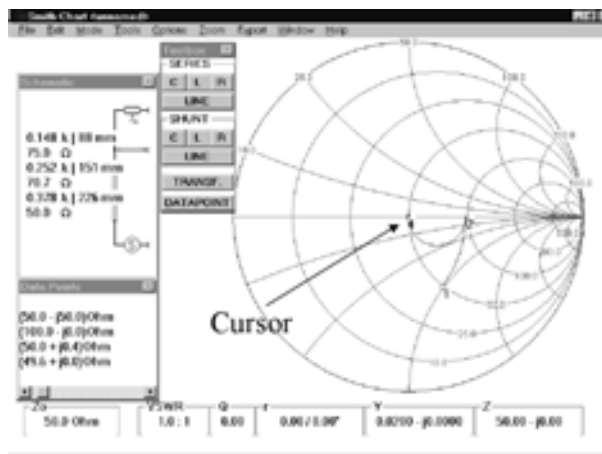


Figure 7: Complex load shunt operation

5. Conclusions

As demonstrated, the advantages of using the Smith Chart are greatly enhanced by the availability of the software employed. The demo version of this software is powerful enough to be employed as an educational tool in explaining the mechanics of the Smith Chart. One advantage of using the PC program is that it streamlines the Smith Chart techniques in terms of speed, accuracy and visual aesthetics. With this program the student can focus on the prediction methodology for the Smith Chart rather than somewhat tedious ruler and compass based exercises.

References

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- [6] W. Tomasi, See Appendix A, *Electronic Communication Systems*, Prentice Hall, Columbus Ohio, 3rd Edition, 1998.
- [7] Details on down loading demo Version 1.91 from internet (www.hta-be.bfh.ch/~dellsper), contact for acquiring the full version.