THE SMITH CHART

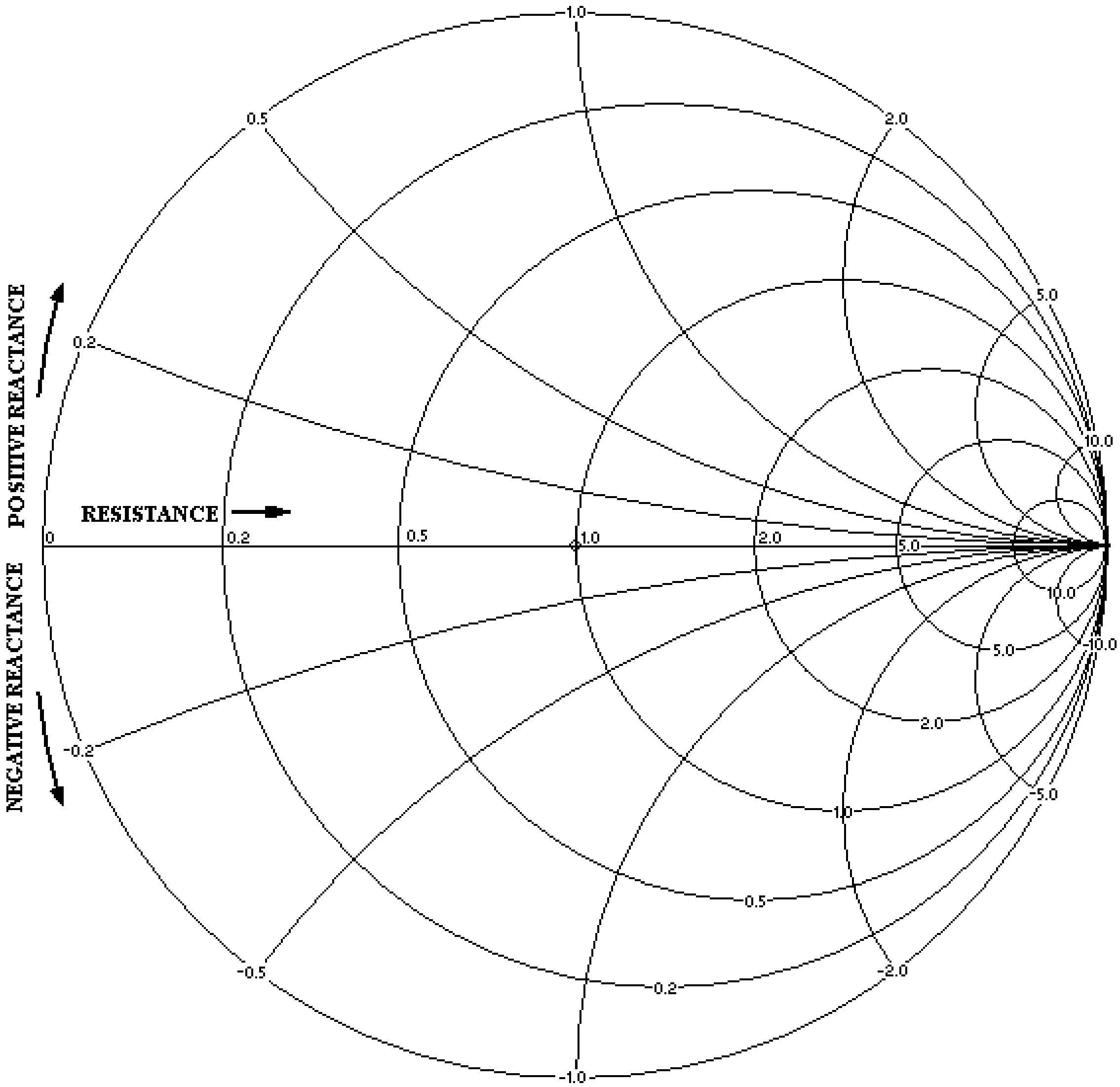
.In the last section we looked at the properties of two particular lengths of resonant transmission lines: half and quarter wavelength lines. It is possible to compute the impedance transformation of an arbitrary length of line by using the following formula:

$$Z_{INPUT} = Z_{LINE}\left[\frac{Z_{LOAD} + jZ_{LINE}\tan\left(\frac{2\pi lf}{984V_F}\right)}{Z_{LINE} + jZ_{LOAD}\tan\left(\frac{2\pi lf}{984\pi V_F}\right)}\right]$$

The lengths are measured in feet, angles in radians, frequencies in MHz and impedances in ohms.

This equation can be cumbersome to solve. All the impedances are often complex numbers, and the formula itself has real and complex parts. A graphical method for solving this equation, known as the Smith Chart, was developed by P.H. Smith in the late 1930's and originally described in Electronics for January1939. It may he obtained at most university book stores and is still used today because of its simplicity.

The following paragraphs explain the theory behind and the use of the Smith Chart. You may want to click here to look at an IEEE paper that explains the Smith Chart in a different way, to improve your understanding



SIMPLIFIED SMITH SHOWING RESISTANCE AND REACTANCE AXES

One of the simpler applications is to determine the feed-point impedance of an antenna, based on an impedance measurement at the input of a random length of transmission line. By using the Smith Chart, the impedance measurement can be made with the antenna in place atop a tower or mast, and there is no need to cut the line to an exact multiple of half wavelengths. The Smith Chart may be used for other purposes, too, such as the design of impedance-matching networks. These matching networks can take on any of several forms, such as L and pi networks, a stub matching system, a series-section match, and more. With a knowledge of the Smith Chart, the technician can eliminate much "cut and try” work.

The input impedance, or the impedance seen when "looking into" a length of transmission line, is dependent upon the SWR, the length of the line, and the characteristic impedance of the line. The SWR, in turn, is dependent upon the load that terminates the line. There are complex mathematical relationships that may be used to calculate the various values of impedances, voltages, currents, and SWR values that exist in the operation of a particular transmission line. These equations can be solved with a personal computer and suitable software, or the parameters maybe determined with the Smith Chart. Even if a computer is used, a fundamental knowledge of the Smith Chart will promote a letter understanding of the problem being solved. And such an understanding might lead to a quicker or simpler solution than otherwise. If the terminating impedance is known, it is a simple matter to determine the input impedance of the line for any length by means of the chart. Conversely, as indicated above, with a given line length and a known (or measured) input impedance, the load impedance may be determined by means of the chart-a convenient method of remotely determining an antenna impedance, for example.

Although its appearance may at first seem somewhat formidable, the Smith Chart is really nothing more than a specialized type of graph. Consider it as having curved, rather than rectangular, coordinate lines. The coordinate system consists simply of two families of circles-the resistance family, and the reactance family. The resistance circles, Fig 1, are centered on the resistance axis (the only straight line on the chart), and are tangent to the outer circle at the right of the chart. Each circle is assigned a value of resistance, which is indicated at the point where the circle crosses the resistance axis. All points along any one circle have the same resistance value.

The values assigned to these circles vary from zero at the left of the chart to infinity at the right, and actually represent a ratio with respect to the impedance value assigned to the center point of the chart, indicated 1.0. This center point is called prime center If prime center is assigned a value of 100 ohms, then 200 ohm resistance is represented by the 2.0 circle, 50 ohms by the 0.5 circle, 20 ohms by the 0.2 circle, and so on. If, instead, a value of 50 is assigned to prime center, the 2.0 circle now represents 100 ohms, the 0.5 circle 25 ohms, and the 0.2 circle 10 ohms. In each case, it may be seen that the value on the chart is determined by dividing the actual resistance by the number assigned to prime center.

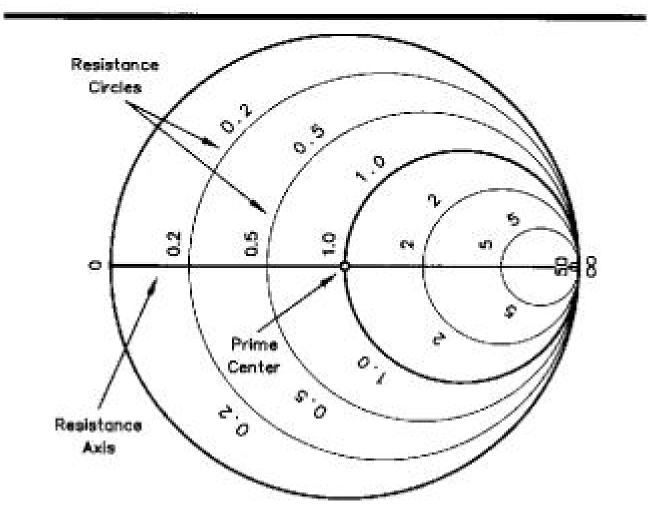


Fig 1—Resistance circles of the Smith Chart coordinate system.

This process is called normalizing.

Conversely, values from the chart are converted back to actual resistance values by multiplying the chart value times the value assigned to prime center. This feature permits the use of the Smith Chart for any impedance values, and therefore with any type of uniform transmission line, whatever its impedance may be. As mentioned above, specialized versions of the Smith Chart may be obtained with a value of 50 ohms at prime center. These are intended for use with 50 ohm lines.

Now consider the reactance circles, Fig 2, which appear as curved lines on the chart because only segments of the complete circles are drawn. These circles are tangent to the resistance axis, which itself is a member of the reactance family (with a radius of infinity). The centers are displaced to the top or bottom on a line tangent to the right of the chart. The large outer circle bounding the coordinate portion of the chart is the reactance axis.

Each reactance circle segment is assigned a value of reactance, indicated near the point where the circle touches the reactance axis. All points along any one segment have the same reactance value. As with the resistance circles, the values assigned to each reactance circle are normalized with respect to the value assigned to prime center Values to the top of the resistance axis are positive (inductive), and those to the bottom of the resistance axis are negative (capacitive).

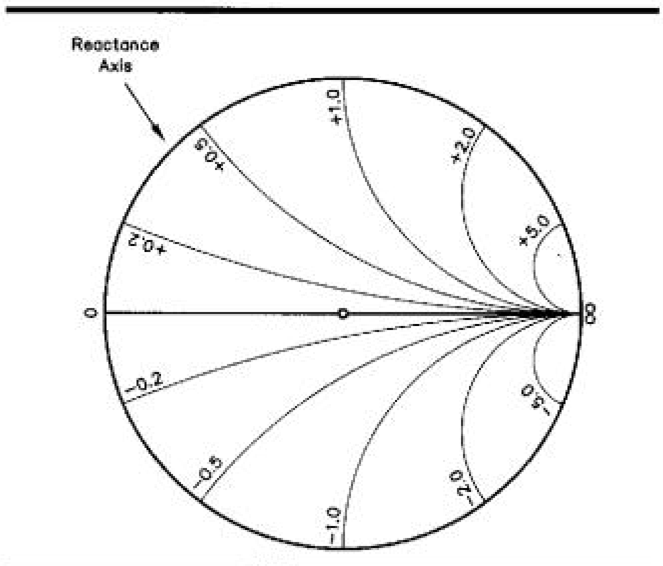


Fig 2—Reactance circles (segments) of the Smith Chart coordinate system.

When the resistance family and the reactance family of circles are combined, the coordinate system of the Smith Chart results, as shown in Fig 3. Complex impedances (R +jX) can be plotted on this coordinate system.

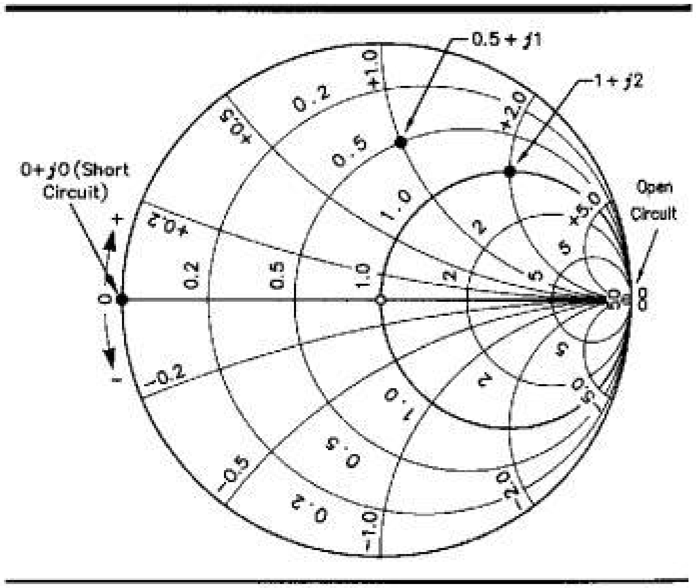


Fig 3—The complete coordinate system of the Smith Chart. For simplicity, only a few divisions are shown for the resistance and reactance values.

2. Impedance Plotting

Suppose we have an impedance consisting of 50 ohms resistance and 100 ohms inductive reactance (Z = 50 +j100 ohms). If we assign a value of 100 ohms to prime center, we normalize the above impedance by dividing each component of the impedance by 100 ohms. The normalized impedance is then 50/100 +j(100/100) = 0.5 +jl .0. This impedance is plotted on the Smith Chart at the intersection of the 0.5 resistance circle and the +1.0 reactance circle, as indicated in Fig 3. Calculations may now be made from this plotted value.

Now say that instead of assigning 100 ohms to prime center, we assign a value of 50 ohms. With this assignment, the 50 +jl100 ohm impedance is plotted at the intersection of the 50/50 = 1.0 resistance circle, and the 100/50 = 2.0 positive reactance circle. This value, 1 + j2, is also indicated in Fig 3. But now we have two points plotted in Fig 3 to represent the same impedance value, 50 +j100 ohms. How can this be?

These examples show that the same impedance may be plotted at different points on the chart, depending upon the value assigned to prime center. But two plotted points cannot represent the same impedance at the same time! It is customary when solving transmission-line problems to assign to prime center a value equal to the characteristic impedance, or Z_0, of the line being used. This value should always be recorded at the start of calculations, to avoid possible confusion later (In using the specialized charts with the value of 50 at prime center, it is, of course, not necessary to normalize impedances when working with 50 ohm line. The resistance and reactance values may he read directly from the chart coordinate system.)

Prime center is a point of special significance. As just mentioned, it is customary when solving problems to assign the Z_0 value of the line to this point on the chart-50 ohms for a 50 ohm line, for example. What this means is that the center point of the chart now represents 50 + j0 ohms-a pure resistance equal to the characteristic impedance of the line. If this were a load on the line, we recognize from transmission-line theory that it represents a perfect match, with no reflected power and with a 1.0 to I SWR. Thus, prime center also represents the 1.0 SWR circle (with a radius of zero). SWR circles are also discussed in a later section.

Short and Open Circuits

On the subject of plotting impedances, two special cases deserve consideration. These are short circuits and open circuits. A true short circuit has zero resistance and zero
reactance, or 0 +j0 ohms). This impedance is plotted at the left of the chart, at the intersection of the resistance and the reactance axes. By contrast, an open circuit has infinite resistance, and therefore is plotted at the right of the chart, at the intersection of the resistance and reactance axes. These two special cases are sometimes used in matching stubs, described later.

Standing-Wave-Ratio Circles

Members of a third family of circles, which are not printed on the chart but which are added during the process of solving problems, are standing-wave-ratio or SWR circles. See Fig 4. This family is centered on prime center, and appears as concentric circles inside the reactance axis. During calculations, one or more of these circles may be added with a drawing compass. Each circle represents a value of SWR, with every point on a given circle representing the same SWR. The SWR value for a given circle may be determined directly from the chart coordinate system, by reading the resistance value where the SWR circle crosses the resistance axis to the right of prime center (The reading where the circle crosses the resistance axis to the left of prime center indicates the inverse ratio.)

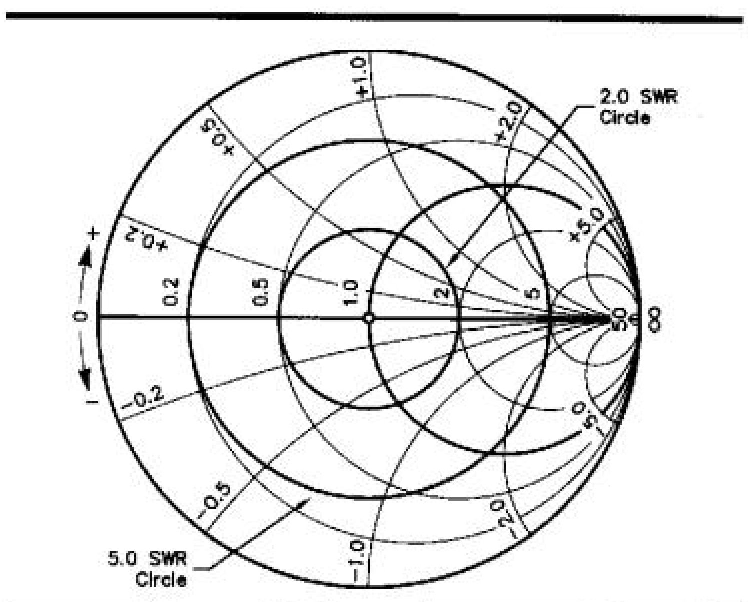


Fig 4—Smith Chart with SWR circles added.

Consider the situation where a load mismatch in a length of line causes a 3-to-i SWR ratio to exist. If we temporarily disregard line losses, we may state that the SWR remains constant throughout the entire length of this line. This is represented on the Smith Chart by drawing a 3:1 constant SWR circle (a circle with a radius of 3 on the resistance axis) as in Fig 5. The design of the chart is such that any impedance encountered anywhere along the length of this mismatched line will fall on the SWR circle. The impedances may be read from the coordinate system merely by the progressing around the SWR circle by an amount corresponding to the length of the line involved.

This brings into use the wavelength scales, which appear in Fig 5 near the perimeter of the Smith Chart. These scales are calibrated in terms of portions of an electrical wavelength along a transmission line. Both scales start from 0 at the left of the chart. One scale, running counterclockwise, starts at the generator or input end of the line and progresses toward the load. The other scale starts at the load and proceeds toward the generator in a clockwise direction. The complete circle around the edge of the chart represents ½ λ. Progressing once around the perimeter of these scales corresponds to progressing along a transmission line for 1/2 λ. Because impedances repeat themselves every ½ λ along a piece of line, the chart may be used for any length of line by disregarding or subtracting from the line's total length an integral. or whole number, of half wavelengths.

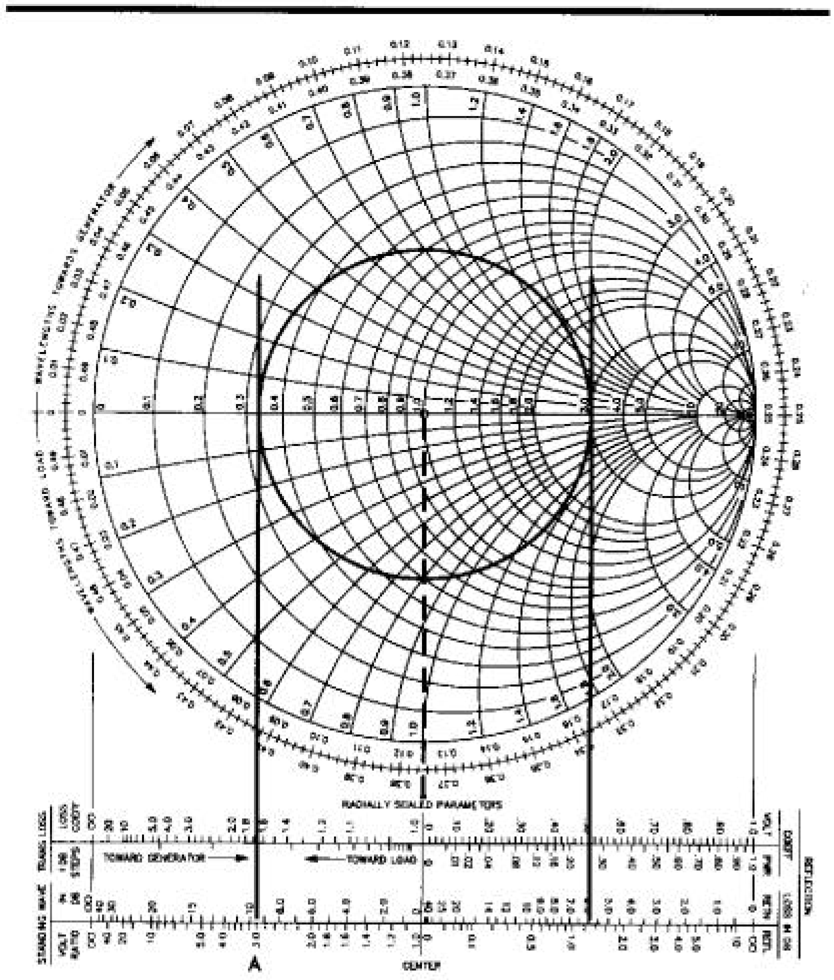


Fig 5—Example discussed in text.

Also shown in Fig 5 is a means of transferring the radius of the SWR circle to the external scales of the chart, by drawing lines tangent to the circle. Another simple way to obtain information from these external scales is to transfer the radius of the SWR circle to the external scale with a drawing compass. Place the point of a drawing compass at the center or 0 line, and inscribe a short arc across the appropriate scale. It will be noted that when this is done in Fig 5, the external STANDING-WAVE VOLTAGE-RATIO scale indicates the SWR to be 3.0 (at A) our condition for initially drawing the circle on the chart (and the same as the SWR reading on the resistance axis).

3. Solving Problems with the Smith Chart

Suppose we have a transmission line with a characteristic impedance of 50 ohms and an electrical length of 0.3λ . Also, suppose we terminate this line with an impedance having a resistive component of 25 ohms and an inductive reactance of 25 ohms (Z = 25 +j25). What is the input impedance to the line?

The characteristic impedance of the line is 50 ohms, so we begin by assigning this value to prime center Because the line is not terminated in its characteristic impedance, we know that standing waves will exist on the line, and that, therefore, the input impedance to the line will not be exactly 50 ohms. We proceed as follows. First, normalize the load impedance by dividing both the resistive and reactive components by 50 ohms (Z_0 of the line being used). The normalized impedance in this case is 0.5 + j0.5. This is plotted on the chart at the intersection of the 0.5 resistance and the +0.5 reactance circles, as in Fig 6. Then draw a constant SWR circle passing through this point. Transfer the radius of this circle to the external scales with the drawing compass. From the external STANDING-WAVE VOLTAGE~RATIO scale, it may be seen (at A) that the voltage ratio of 2.62 exists for this radius, indicating that our line is operating with an SWR of 2.62 to 1. This figure is convened to decibels in the adjacent scale, where 8.4 dB may be read (at B), indicating that the ratio of the voltage maximum to the voltage minimum along the line is 8.4 dB. (This is mathematically equivalent to 20 times the log of the SWR value.)

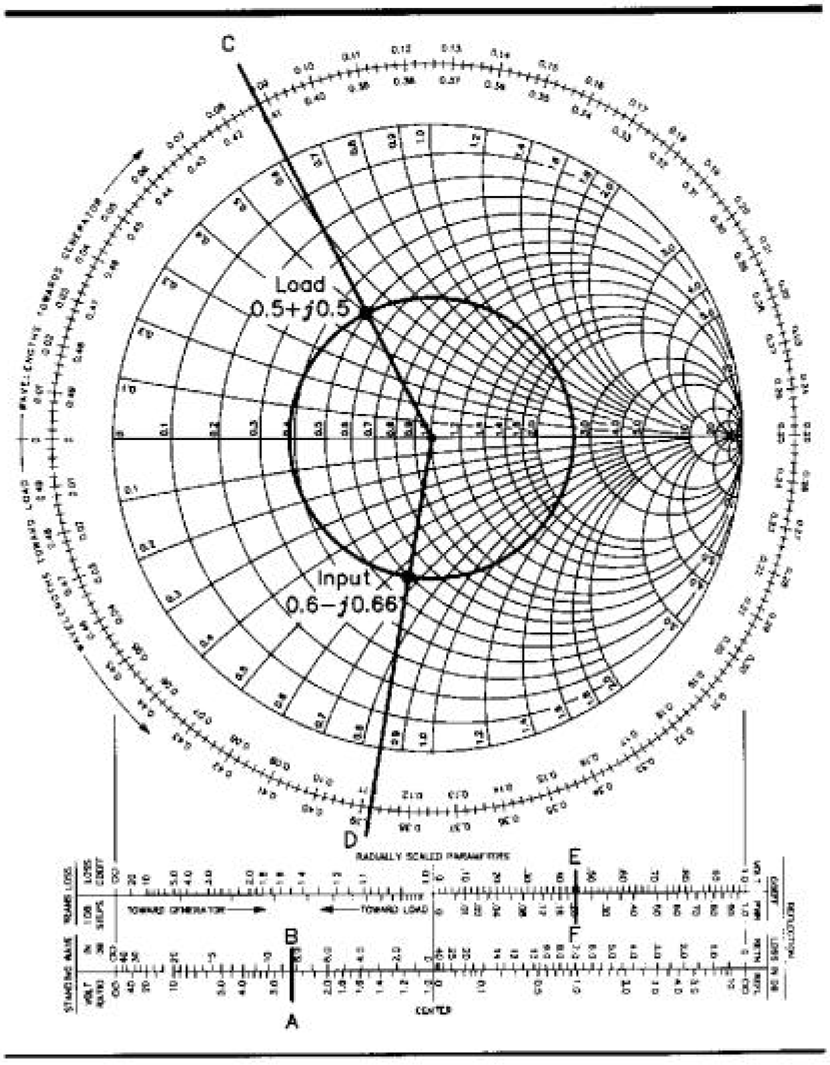


Fig 6—Example discussed in text.

Next, with a straightedge, draw a radial line from prime center through the plotted point to

intersect the wavelengths scale. At this intersection, point C in Fig 6, read a value from the wavelengths scale. Because we are starting from the load, we use the TOWARD GENERATOR or outermost calibration, and read 0.088 λ. To obtain the line input impedance, we merely find the point on the SWR circle that is 0.3 A toward the generator
from the plotted load impedance. This is accomplished by adding 0.3 (the length of the line in wavelengths) to the reference or starting point, 0.088, 0.3 + 0.088 = 0.388.
Locate 0.388 on the TOWARD GENERATOR scale (at D). Draw a second radial line from this point to prime center. The intersection of the new radial line with the SWR circle represents the normalized line input impedance, in this case 0.6 –j0.66.

To find the unnormalized line impedance, multiply by 50, the value assigned to prime center. The resulting value is 30 -j33, or 30 ohms resistance and 330 capacitive reactance. This is the impedance that a transmitter must match if such a system were a combination of antenna and transmission line. This is also the impedance that would be measured on an impedance bridge if the measurement were taken at the line input.

In addition to the line input impedance and the SWR, the chart reveals several other operating characteristics of the above system of line and load, if a closer look is desired. For example, the voltage reflection coefficient, both magnitude and phase angle, for this particular load is given. The phase angle is read under the radial line drawn through the plot of the load impedance, where the line intersects the ANGLE OF REFLECTION COEFFICIENT scale. This scale is not included in Fig 6, but will be found on the Smith Chart just inside the wavelengths scales. In this example, the reading is 116.6 degrees. This indicates the angle by which the reflected voltage wave leads the incident wave at the load. It will be noted that angles on the bottom half, or capacitive-reactance half, of the chart are negative angles, a “negative" lead indicating that the reflected voltage wave actually tags the incident wave.

The magnitude of the voltage-reflection-coefficient may be read from the external REFLECTION COEFFICIENT VOLTAGE scale, and is seen to be approximately 0.45 (at E) for this example. This means that 45 percent of the incident voltage is reflected. Adjacent to this scale on the POWER calibration, it is noted (at F) that the power reflection coefficient is 0.20, indicating that 20 percent of the incident power is reflected.
(The amount of reflected power is proportional to the square of the reflected voltage.)

And yet more information on the Smith Chart

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