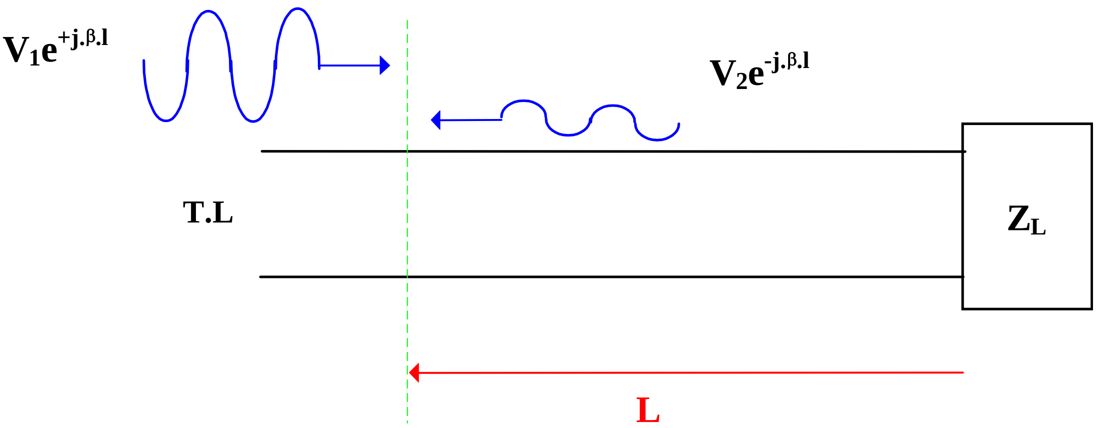
Smith Chart Tutorial Part1

To begin with we start with the definition of VSWR, which is the ratio of the reflected voltage over the incident voltage. The Reflection coefficient Γ is simply the complex (ie has phase) version of VSWR:-

Define voltage standing wave ratio (VSWR)

$$\frac{V_{max}}{V_{min}} = \frac{|V_1| + |V_2|}{|V_1| - |V_2|}$$

Voltage reflection coefficient Γ - Complex

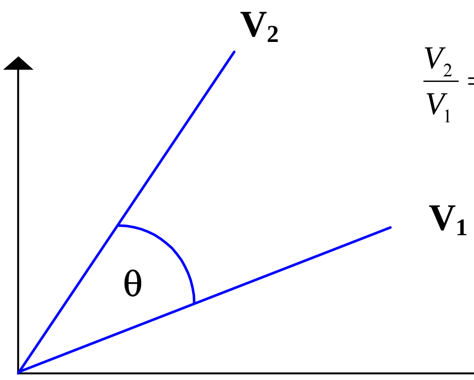


$$\Gamma = \frac{V_2 e^{-j.\beta.\ell}}{V_1 e^{+j.\beta.\ell}} = \frac{V_2}{V_1} e^{-j2\beta.\ell}$$

$$\text{At the load } \Gamma \ (l = 0) \ ; \ \Gamma = \frac{V_2}{V_1} \Rightarrow$$

but this may be complex number if there is an instantaneous phase change which we'll call (ϕ) on reflection.

Phase Diagram



$$\frac{V_2}{V_1} = \left|\frac{V_2}{V_1}\right| e^{j\varnothing} \quad \text{.....(a)} \ \ L = 0$$

$$\text{At L} > 0\ \Gamma_{(L)} = \Gamma_{(0)} e^{-j.\phi} = \frac{|V_2|}{|V_1|} e^{j(\phi - 2\beta\ell)}$$

$$\text{For a lossless line } |V_1|\ \&\ |V_2| \text{ do not vary with L} \therefore |\Gamma| \text{ is constant} = \left|\frac{V_2}{V_1}\right|$$

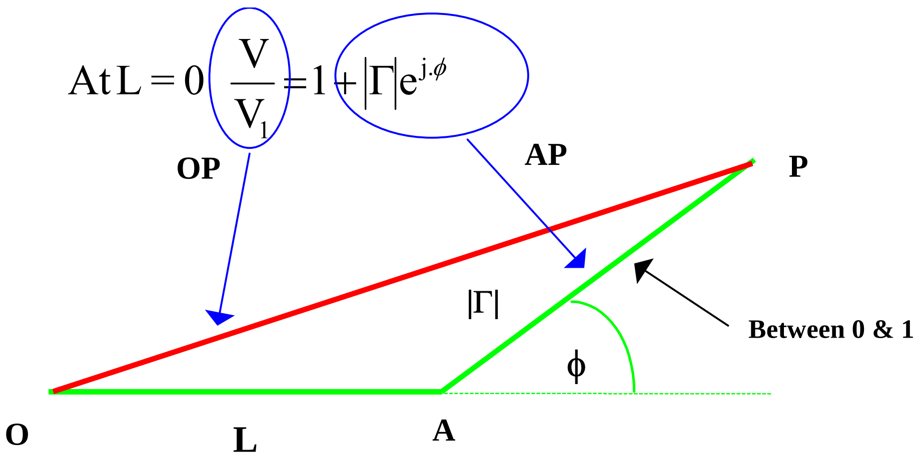
$$\Gamma = |\Gamma| e^{j(\phi - 2\beta.\ell)} \quad \text{represented on Crank Diagram}$$

Crank Diagram

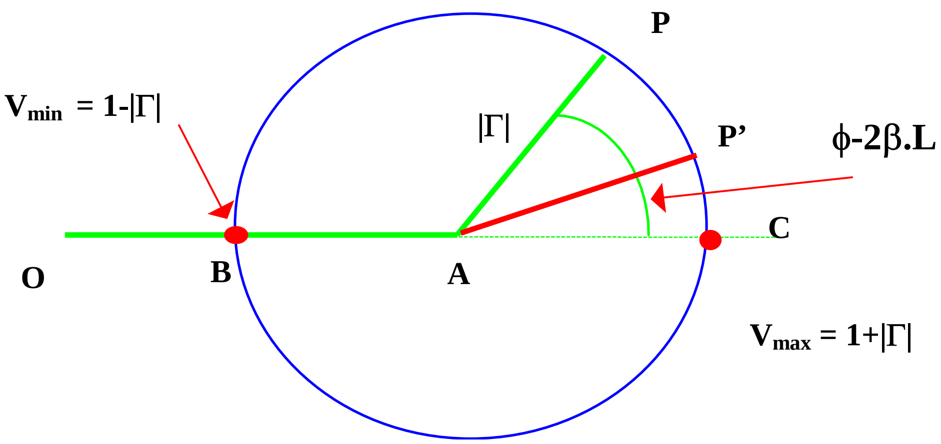
We use a crack diagram as a way of representing the reflection coefficient phasor.

$$V = V_1 e^{+j.\beta.\ell} + V_2 e^{-j.\beta.\ell}$$

$$\therefore \frac{V}{V_1 e^{+j.\beta.\ell}} = 1 + \frac{V_2}{V_1} e^{-j2\beta.\ell} = 1 + |\Gamma| e^{j(\phi - 2\beta.\ell)}$$

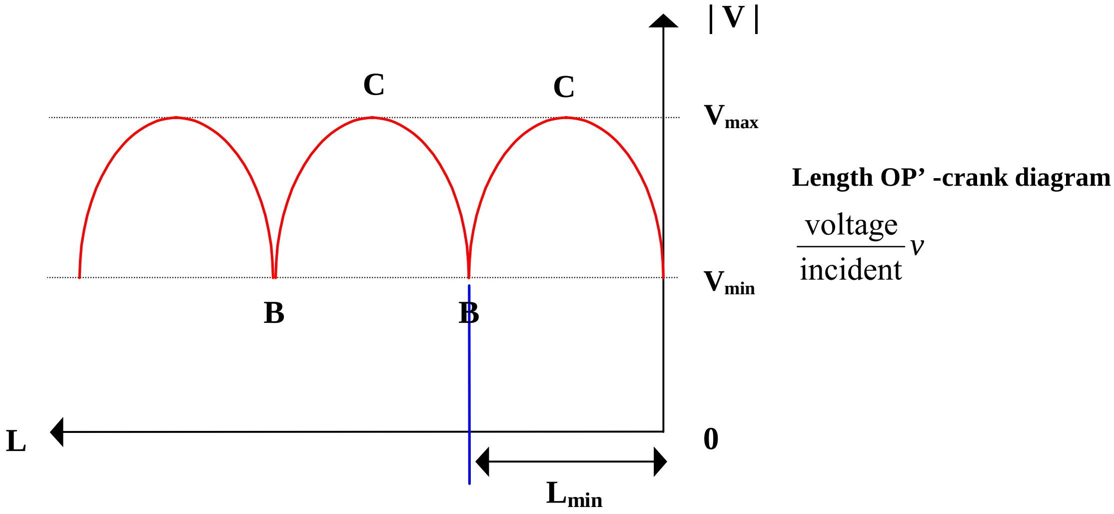


At the origin of argand diagram.OP = magnitude of total voltage/incident voltage



As we saw previously the crack diagram with a circle drawn between points A & C is the beginnings of a Smith chart less the constant resistance and reactance circles/lines.

$$\text{VSWR} = \frac{V_{max}}{V_{min}} = \frac{1+|\Gamma|}{1-|\Gamma|} \text{ or } \Gamma = \frac{\text{VSWR}-1}{\text{VSWR}+1}$$



$$\text{At B } \phi - 2\beta.\ell_{min} = -\pi$$

$$\phi = 2\beta.\ell_{min} - \pi = \frac{4\pi.\ell_{min}}{\lambda_g} - \pi = \phi$$

∴From standing wave pattern measure VSWR ⇒ $|\Gamma|$ @ l_{min} ⇒ ϕ at load.

SmithChart - Impedance (Z) or Admittance Y chart

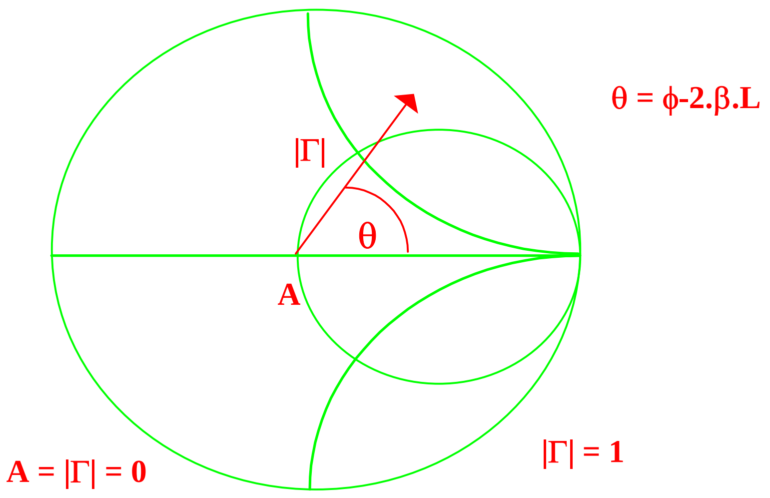
(1) Crank diagram + constant resistance & constant reactance circles.

(2) Graphical solution to the equation

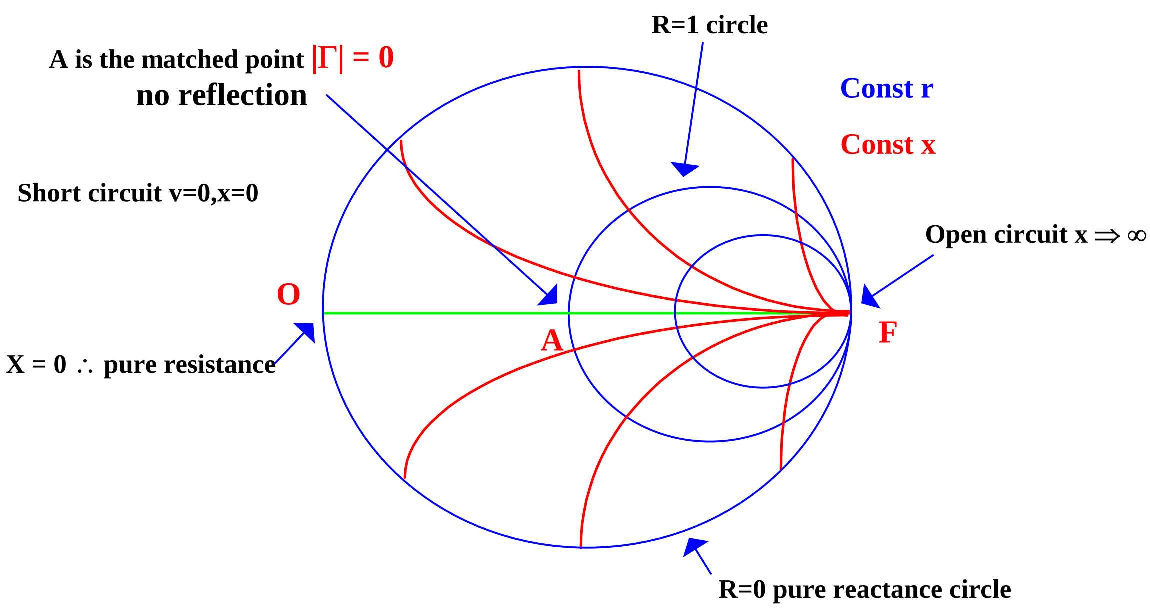
$$\frac{Z_{(in)}}{Z_o} = \frac{1+\Gamma}{1-\Gamma} \qquad \Gamma = |\Gamma| e^{j(\phi - 2\beta.\ell)}$$

complex

(3) Smith Chart is a reflection coefficient diagram



Smith Chart

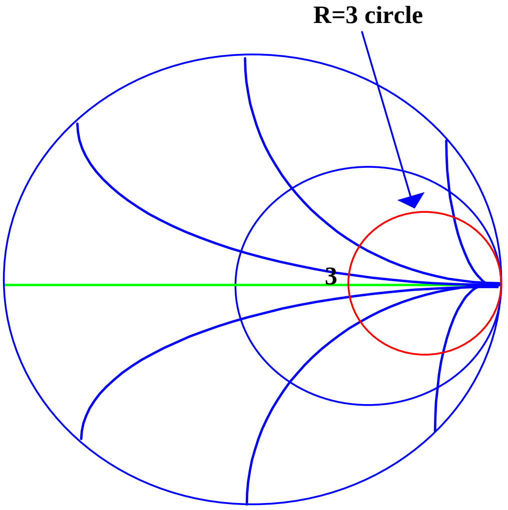


Impedance is plotted on the smith chart by first normalising to the characteristic impedance of the system (usually 50 ohms). In a 50 ohm system the centre of the smith chart is a pure 50 ohms.

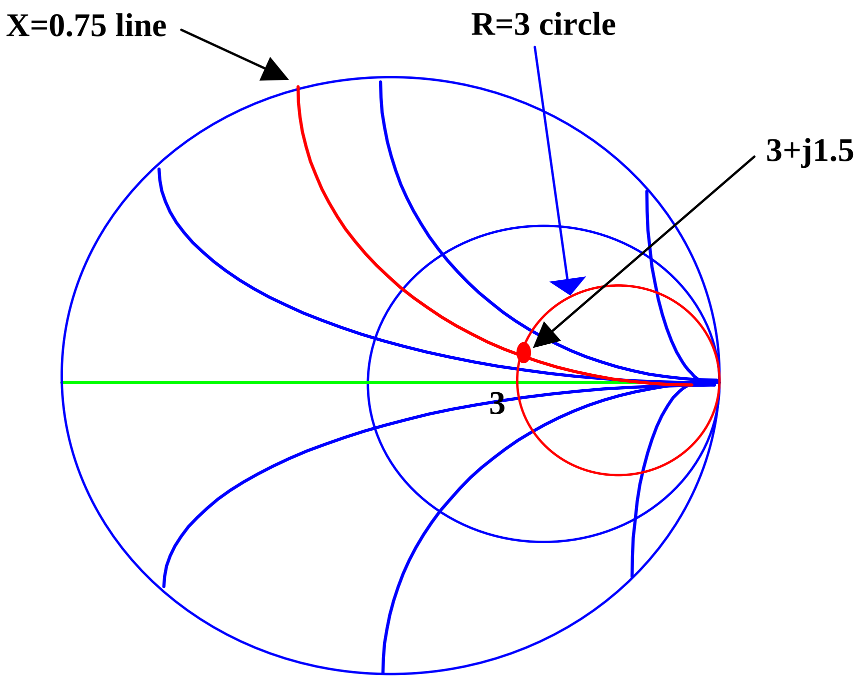
For example say we wanted to plot an impedance of 150 + j75Ω

First normalise ie 150/50 = 3Ω ; 75/50 = 1.5Ω normalised impedance = 3+ j1.5Ω

So the real part of the impedance will lie somewhere along the r = 3 constant resistance circle ie:-

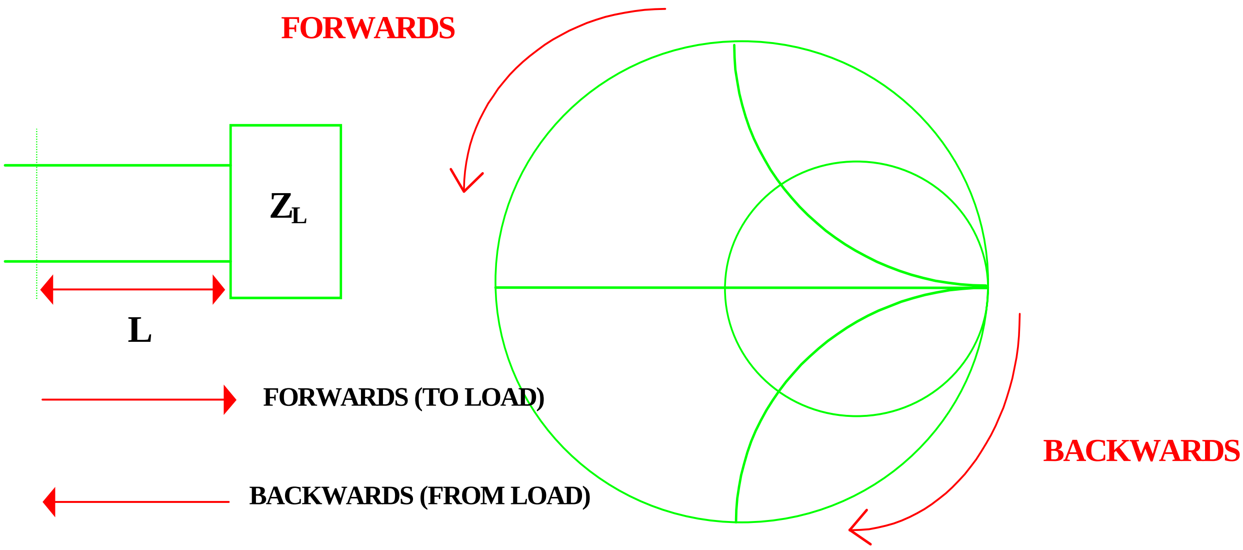


Next we follow the constant reactance line at 0.75 to find the intersection of the r = 3 circle to get to our impedance point.



Using the Smith Chart

(1) Moving along the T.L = rotating around the Smith Chart.



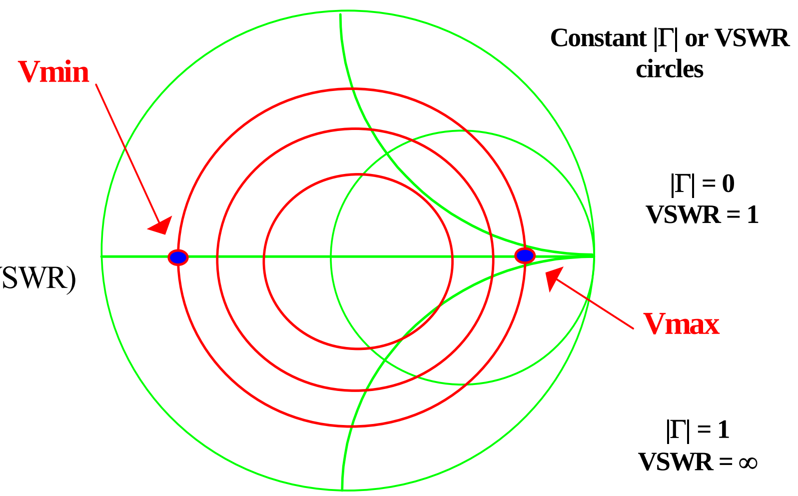
(2) Constant |Γ| or VSWR circles

For a lossless line |Γ| & VSWR do not vary with L.

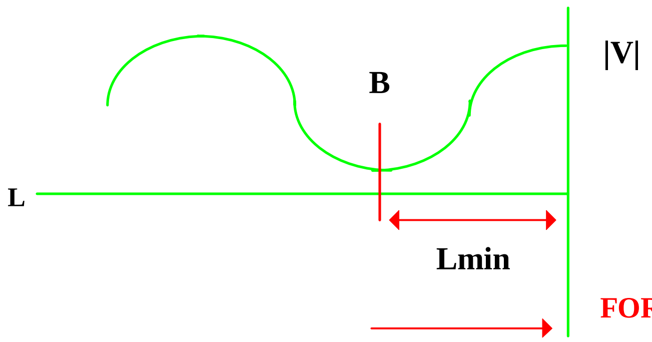
$$\text{VSWR} = \frac{1+|\Gamma|}{1-|\Gamma|} \rightarrow S$$

$$\text{Vmax} = \frac{Z(\max)}{Zo} = S \text{ (real VSWR)}$$

$$\text{Vmin} = \frac{Z(\min)}{Zo} = \frac{1}{S}$$



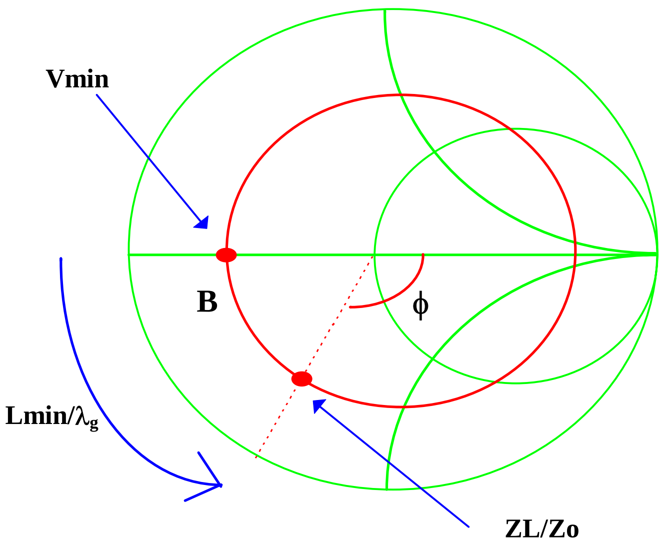
(3) Measure Lmin/λ_g ..... determines ϕ (at load).



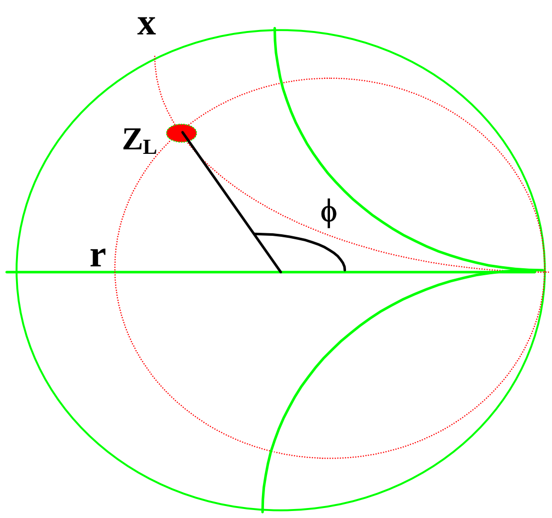
$$\phi = 4\pi \frac{\text{lmin}}{\lambda_g} - \pi$$

angle of reflection coefficient

FORWARD by Lmin/λ_g takes us the load.

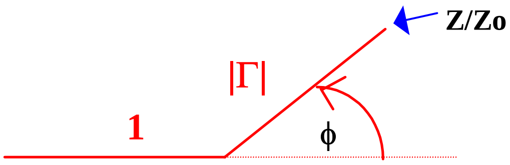


(4) Reading Z from chart also can get |Γ| & ϕ

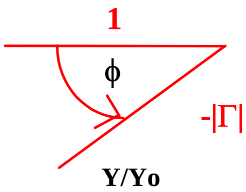


(5)

$$\frac{Z}{Z_o} = \frac{1+\Gamma}{1-\Gamma}$$



Admittance = Y/Y_0 =



On a Smith chart point diametrically opposite $\frac{Z}{Z_o}$ gives $\frac{Y}{Y_o}$

Note $Y_0 = \frac{1}{Z_o}$

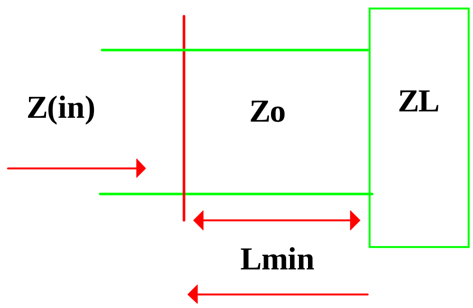
$$Y = G + j.\beta$$

Conductance Susceptance

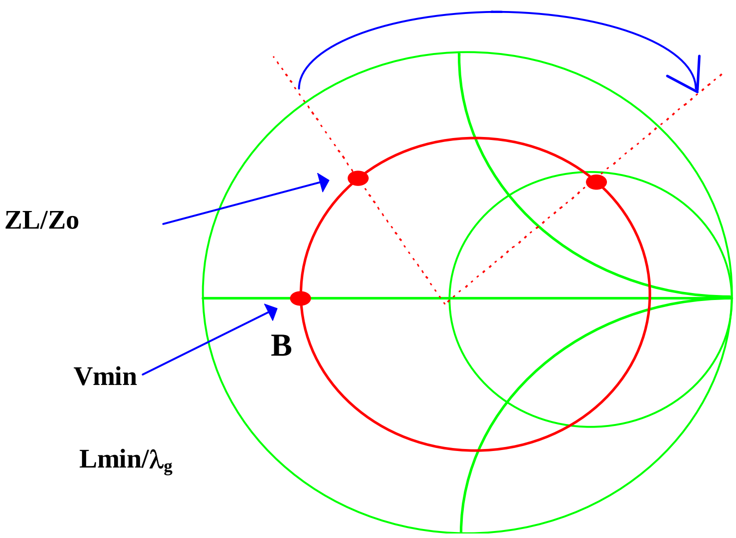
On admittance chart r circles $\rightarrow$ g circles &
x circles $\rightarrow$ b circles.

Note $g = \frac{G}{Y_o}$ and $b = \frac{B}{Y_o}$

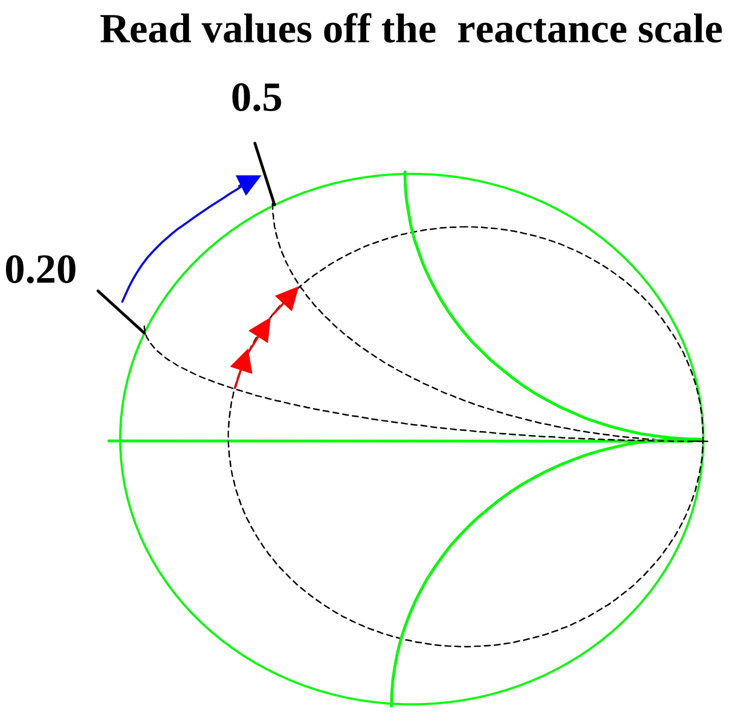
(6) To transform an impedance along a T.L, rotate around the VSWR circle:-



BACKWARDS by Lmin/λ_g takes us to Zin.



(7) Represent a series inductance on a smith chart.



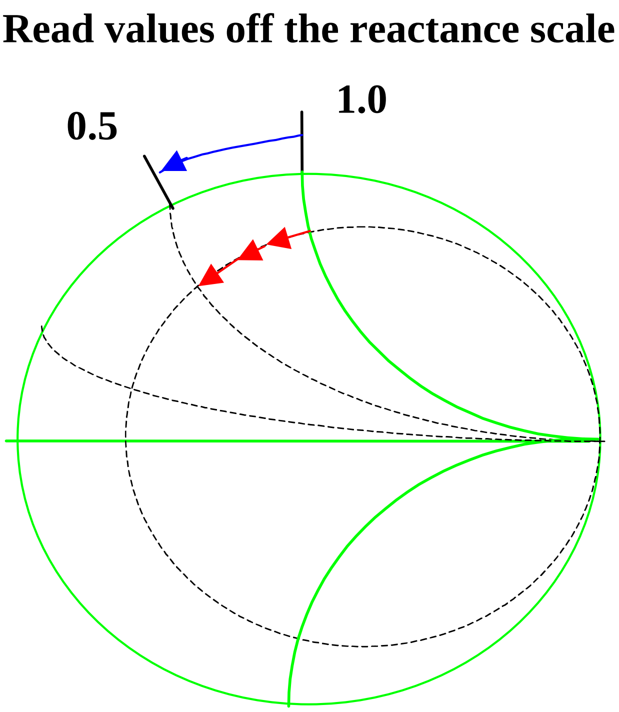
Therefore, assuming a frequency of say 1GHz the value of series inductance represented on the above Smith Chart is given by:-

Reactance (X_L) read from Smith chart = 0.5 - 0.2 Ω = 0.3Ω wrt 50Ω

$$L = \frac{N.X_L}{2\pi f} = \frac{50 * 0.3}{2\pi * 1E^9} = 2.38\text{nH}$$

Similarly for a series capacitor

(8) Represent a series capacitance on a smith chart.



Therefore, assuming a frequency of say 1GHz the value of series capacitance represented on the above Smith Chart is given by:-

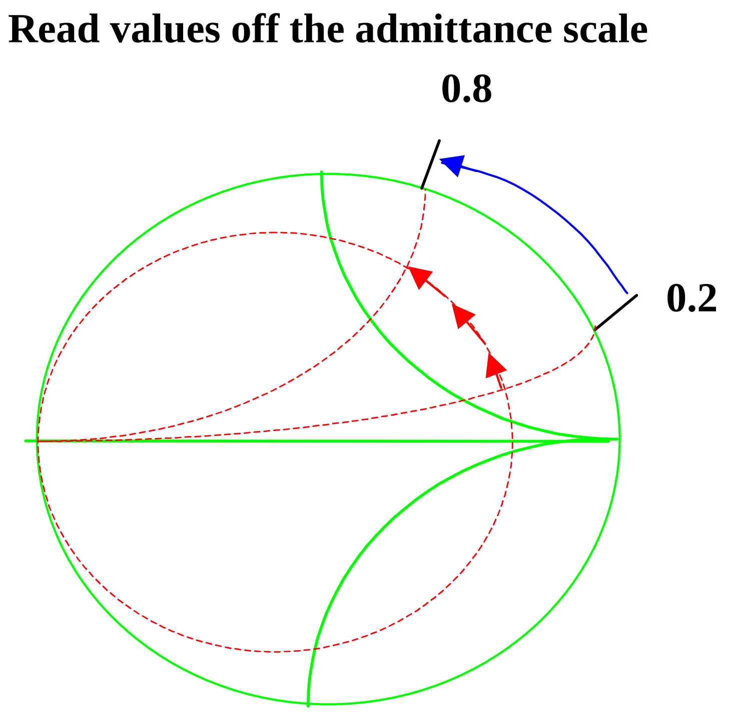
Reactance(X_C) read from Smith chart = 1.0 - 0.5 Ω = 0.5Ω w.r.t 50Ω

$$C = \frac{1}{2\pi f.N.X_C} = \frac{1}{2\pi * 1E^9 * 50 * 0.5} = 6.36pF$$

Where N is the normalising factor (usually 50 ohms)

To represent shunt reactance we need to plot admittance onto the Smith Chart. It is easiest to use a Smith chart with both impedance (usually in black) lines and admittance lines (usually in red) on the same chart. Or you can rotate the Smith chart 180 degrees.

(9) Represent a shunt inductance on a smith chart.



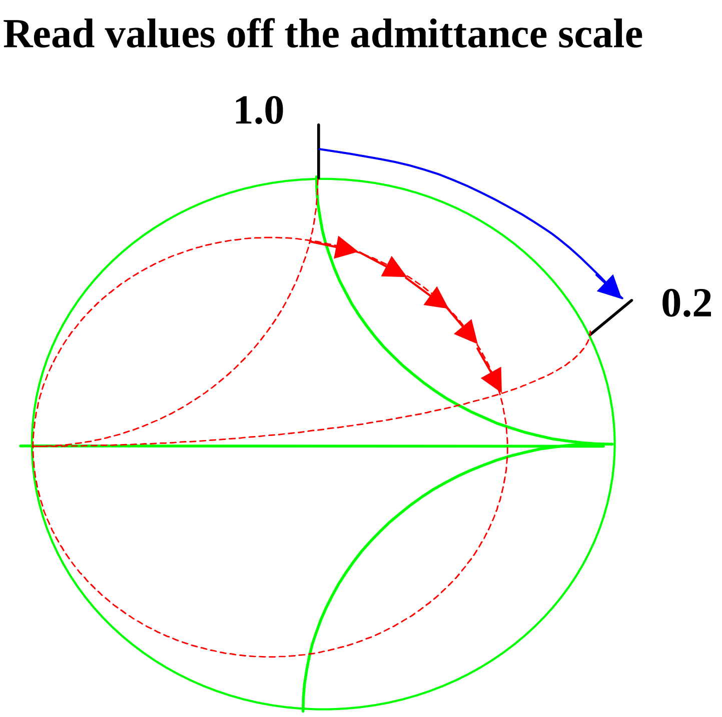
Therefore, assuming a frequency of say 1GHz the value of shunt inductance represented on the above Smith Chart is given by:-

$$\text{Admittance}(Y_L)\ \text{read from Smith chart} = (0.8 - 0.2)\Omega \ = \ 0.6\text{mhos w.r.t } 50\Omega$$

$$L = \frac{N}{2\pi f * Y_L} = \frac{50}{2\pi * 1E^9 * 0.6} = 13.26\text{nH}$$

$$N = \text{normaisation factor (usually 50 ohms)}$$

(10) Represent a shunt capacitance on a smith chart.



Therefore, assuming a frequency of say 1GHz the value of shunt inductance represented on the above Smith Chart is given by:-

$$\text{Admittance}\,(Y_C)\ \text{read from Smith chart} = (1.0 - 0.2)\Omega \;=\; 0.8\text{mhos w.r.t } 50\Omega$$

$$C = \frac{Y_C}{2\pi f * N} = \frac{0.8}{2\pi * 1E^9 * 50} = 2.5\text{pF}$$

N = normaisation factor (usually 50 ohms)